Matematyczne obrazki

Mikołaj Mijakowski

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Przestrzeń ekranu

$$f : \mathbb{R}^2 \times \mathbb{R}^3 \to \mathbb{R}^3$$

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$$f(u, v, r, g, b) = \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

Gradient liniowy

$$\text{clamp}(t) = \begin{cases} 0 & ; \quad t < 0 \\ t & ; \quad t \in [0,1] \\ 1 & ; \quad t > 1 \end{cases}$$

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$$\text{gradient}(\mathbf{x},\mathbf{s},\mathbf{e}) = (1 - \text{clamp}\left(\frac{\langle \mathbf{x}-\mathbf{s}, \mathbf{e}-\mathbf{s}\rangle}{\langle \mathbf{s},\mathbf{s}\rangle}\right)) \begin{bmatrix} R_s \\ B_s \\ G_s \end{bmatrix} + \\ + \text{clamp}\left(\frac{\langle \mathbf{x}-\mathbf{s}, \mathbf{e}-\mathbf{s}\rangle}{\langle \mathbf{s},\mathbf{s}\rangle}\right) \begin{bmatrix} R_e \\ B_e \\ G_e \end{bmatrix}$$



Funkcja smoothstep

$$\text{smoothstep}(t) = \begin{cases} 0 \ ; & t < 0 \\ 3t^2 - 2t^3 \ ; & t \in [0,1] \\ 1 \ ; & t > 1 \end{cases}$$

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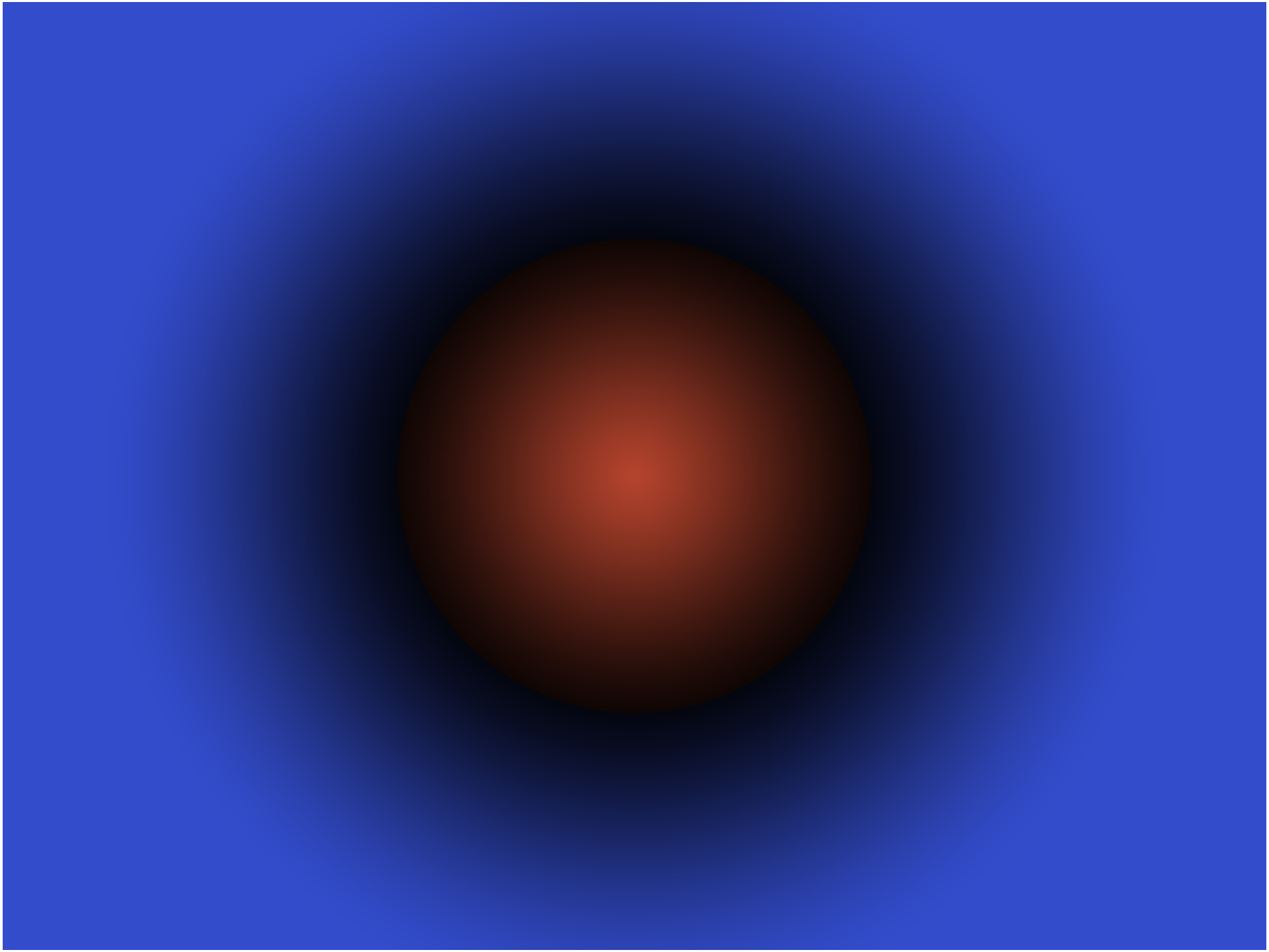
$$\text{gradient}(\mathbf{x},\mathbf{s},\mathbf{e}) = (1 - \text{smoothstep}\left(\frac{\langle \mathbf{x}-\mathbf{s}, \mathbf{e}-\mathbf{s}\rangle}{\langle \mathbf{s},\mathbf{s}\rangle}\right)) \begin{bmatrix} R_s \\ B_s \\ G_s \end{bmatrix} +$$

$$+\text{smoothstep}\left(\frac{\langle \mathbf{x}-\mathbf{s}, \mathbf{e}-\mathbf{s}\rangle}{\langle \mathbf{s},\mathbf{s}\rangle}\right) \begin{bmatrix} R_e \\ B_e \\ G_e \end{bmatrix}$$



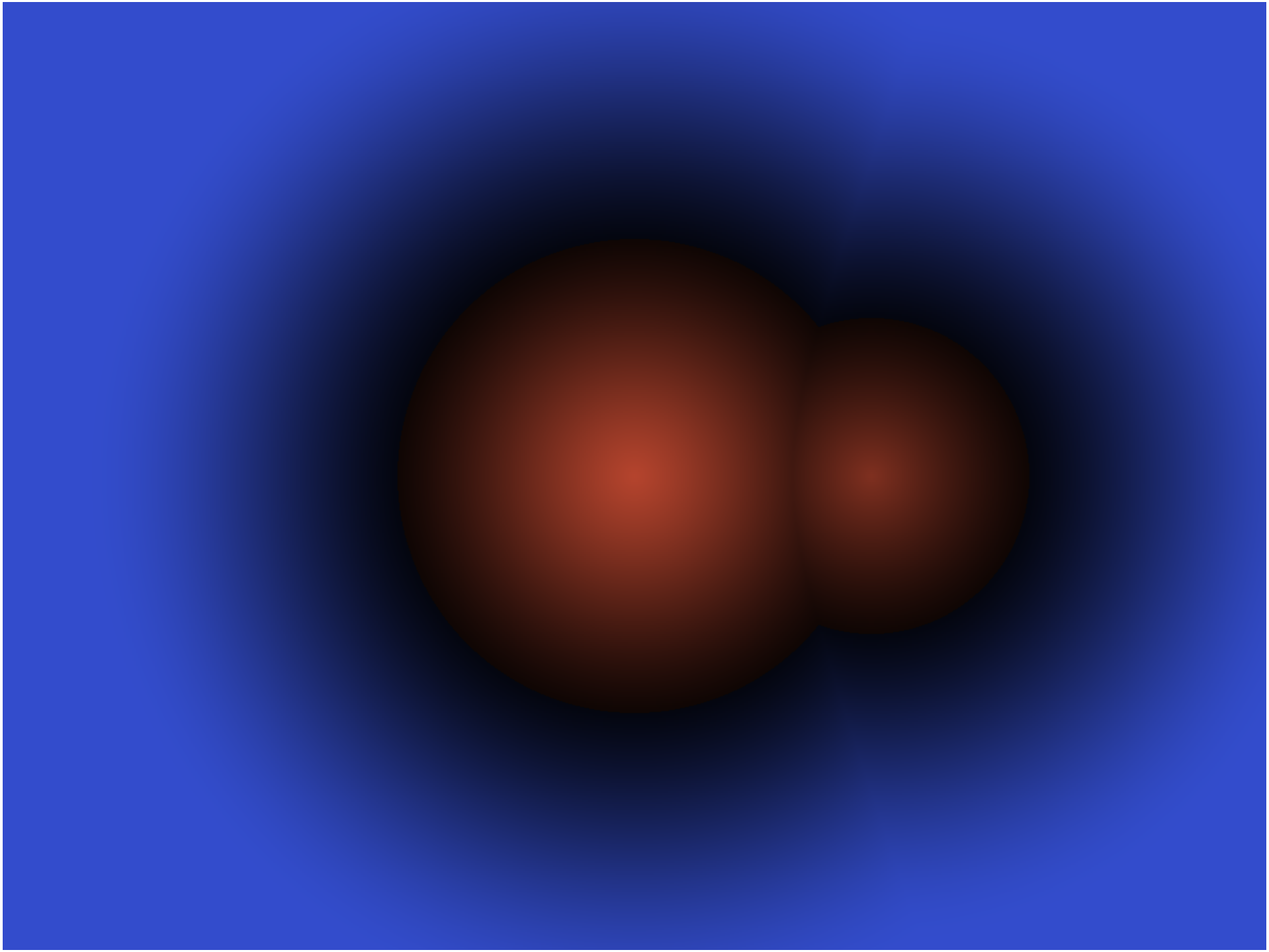
Signed Distance Field

$$\mathrm{sdf}_A(\mathbf{x}) = \mathrm{dist}(\mathbf{x}, A)$$



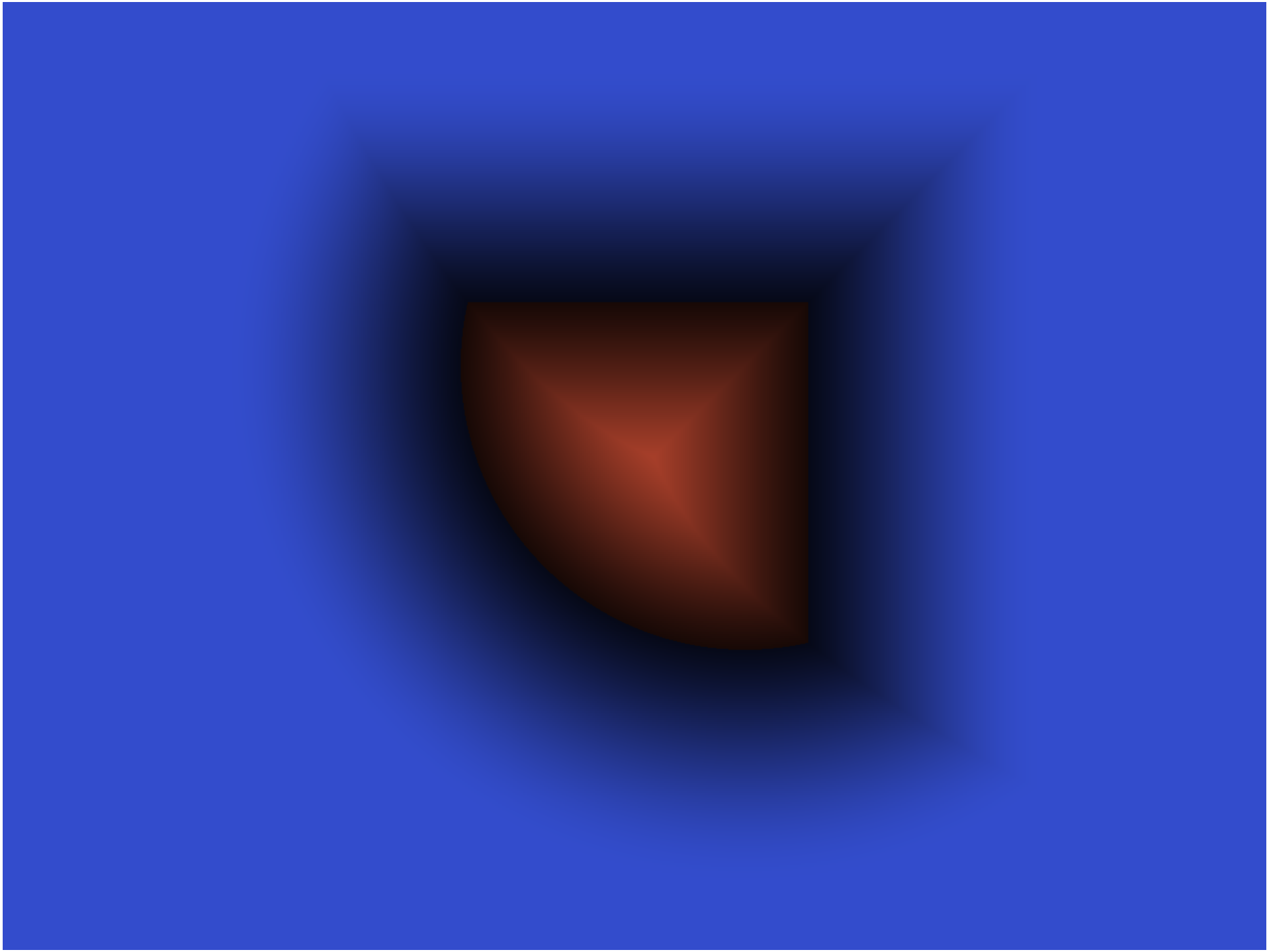
SDF suma

$$\mathrm{sdf}_{A \cup B}(\mathbf{x}) = \min(\mathrm{sdf}_A(\mathbf{x}), \mathrm{sdf}_B(\mathbf{x}))$$



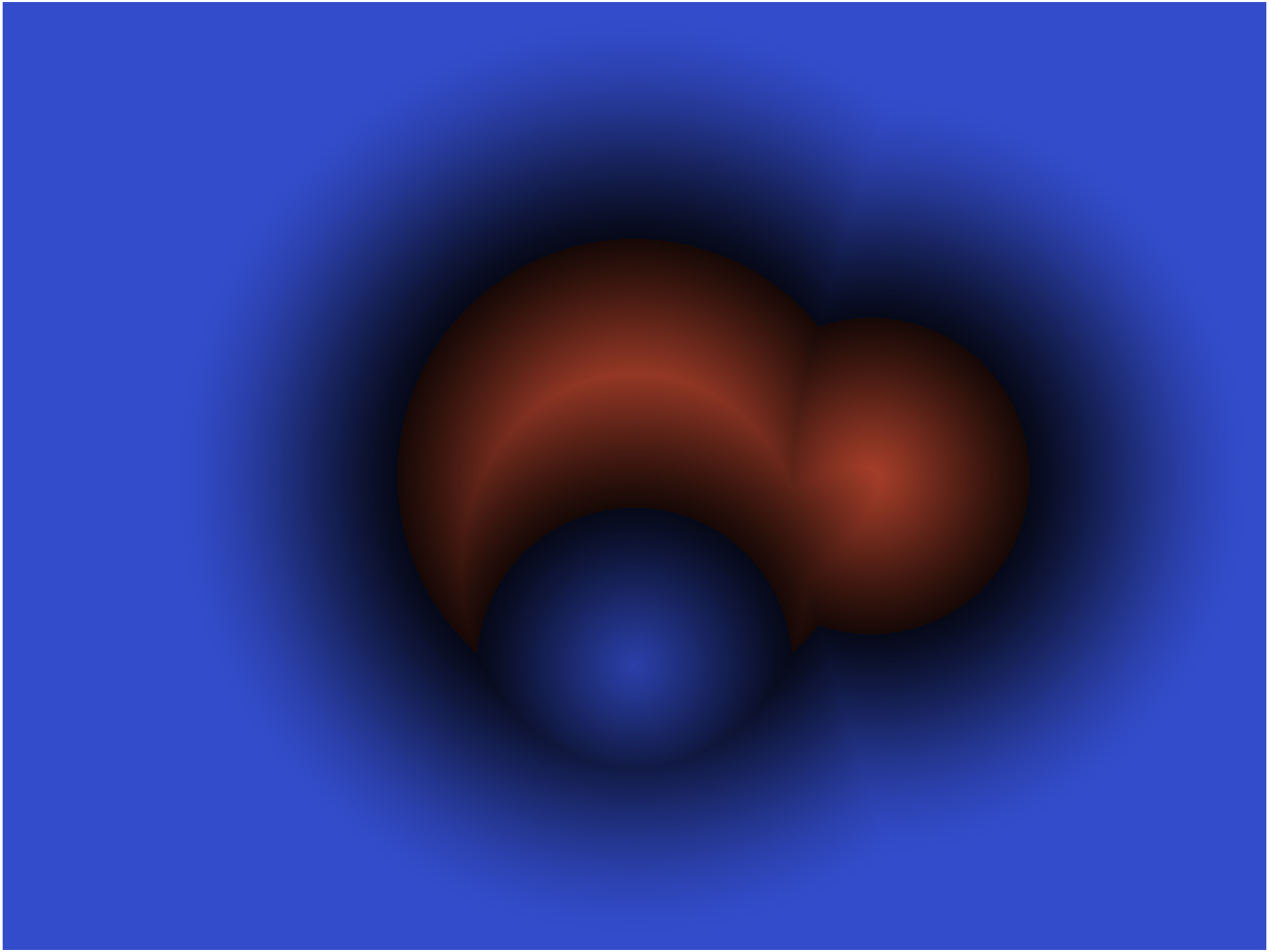
SDF przecięcie

$$\mathrm{sdf}_{A\cap B}(\mathbf{x}) \leqslant \max(\mathrm{sdf}_A(\mathbf{x}), \mathrm{sdf}_B(\mathbf{x}))$$



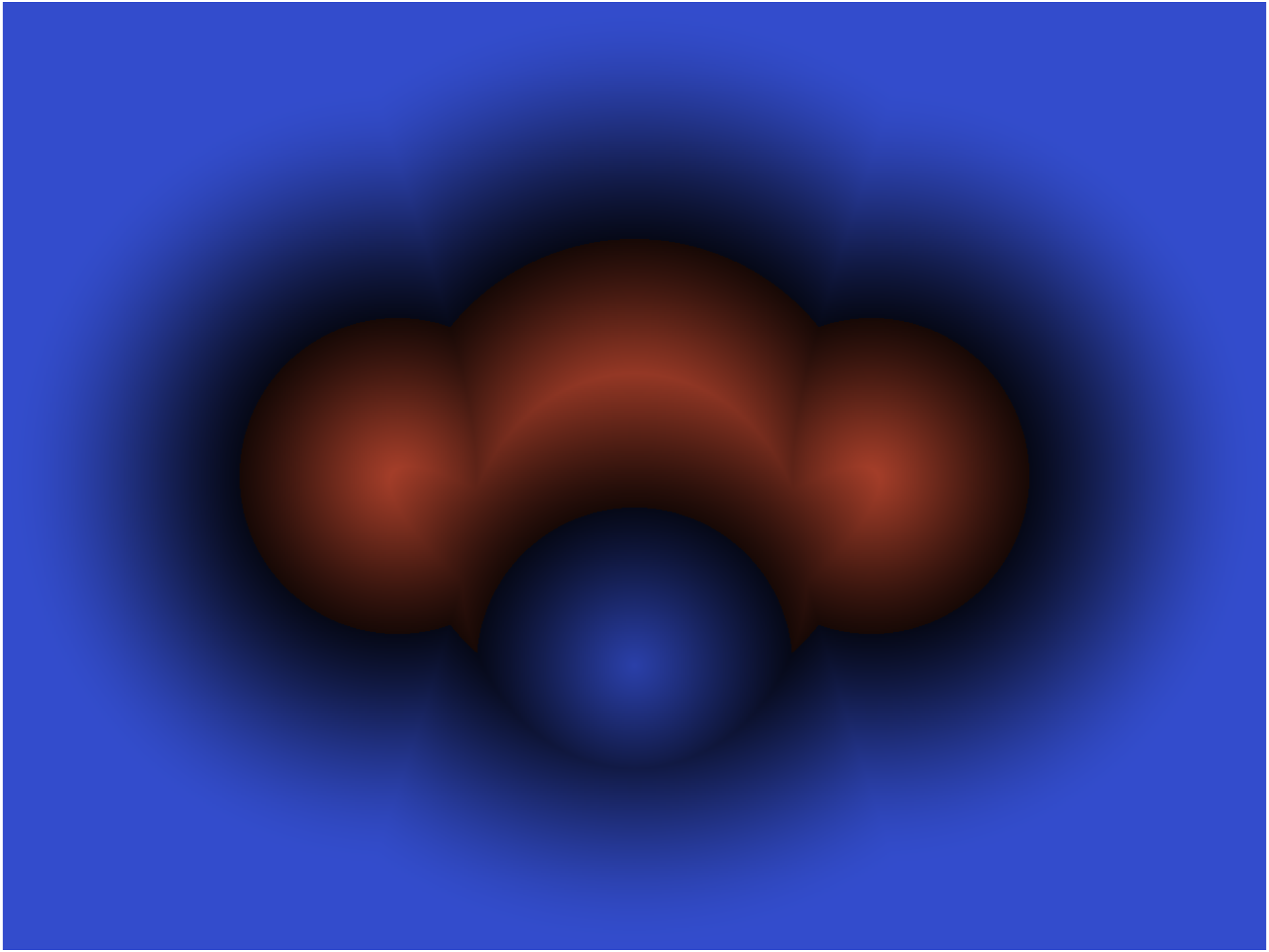
SDF różnica

$$\mathrm{sdf}_{A \setminus B}(\mathbf{x}) \leqslant \max(\mathrm{sdf}_A(\mathbf{x}), -\mathrm{sdf}_B(\mathbf{x}))$$



SDF symetria

$$\mathrm{sdf}_{\mathrm{sym}(A)}(x, y) = \mathrm{sdf}_A(|x|, y)$$



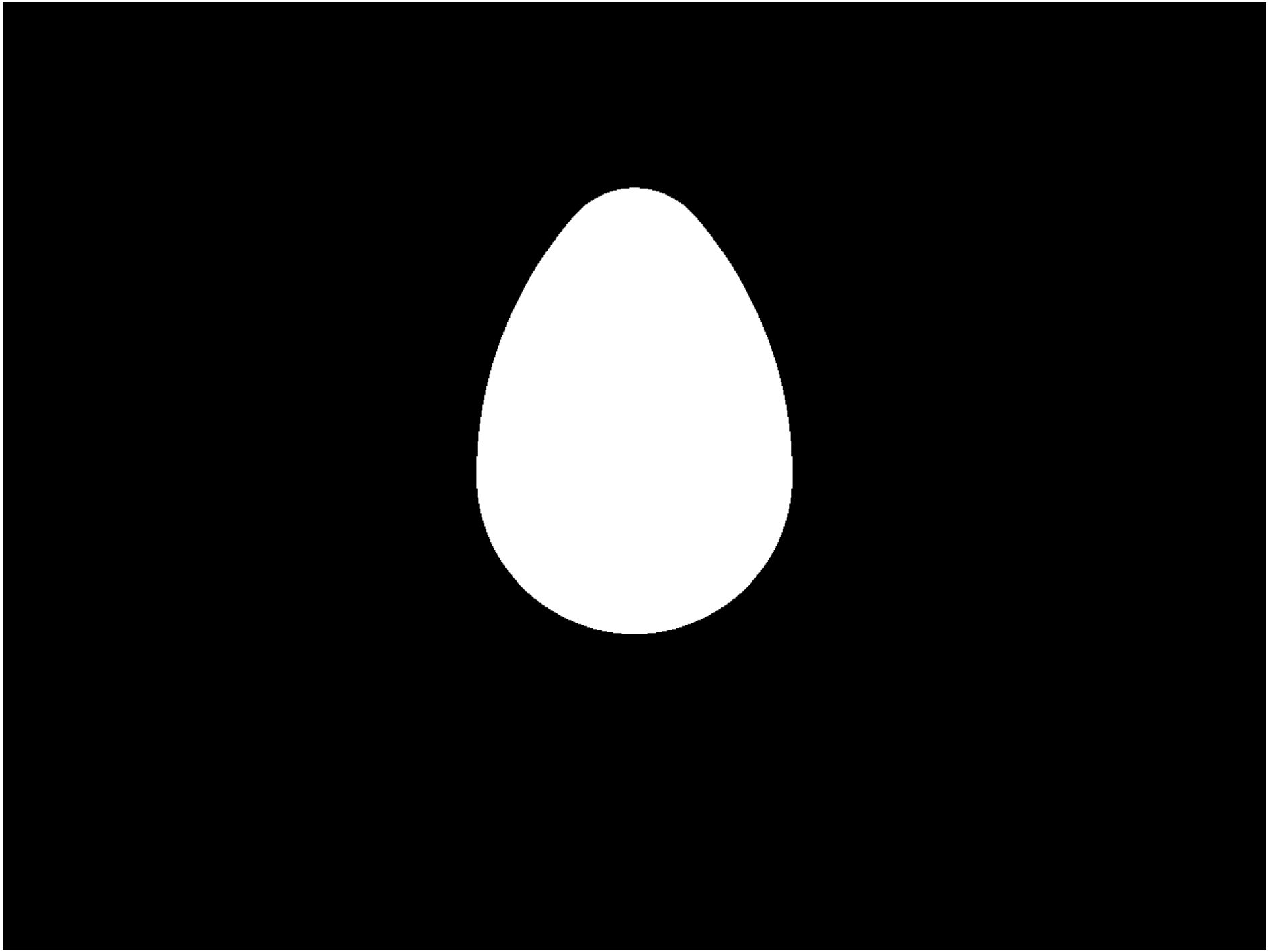
Outline

$$|\mathrm{sdf}(\mathbf{x})| - d$$



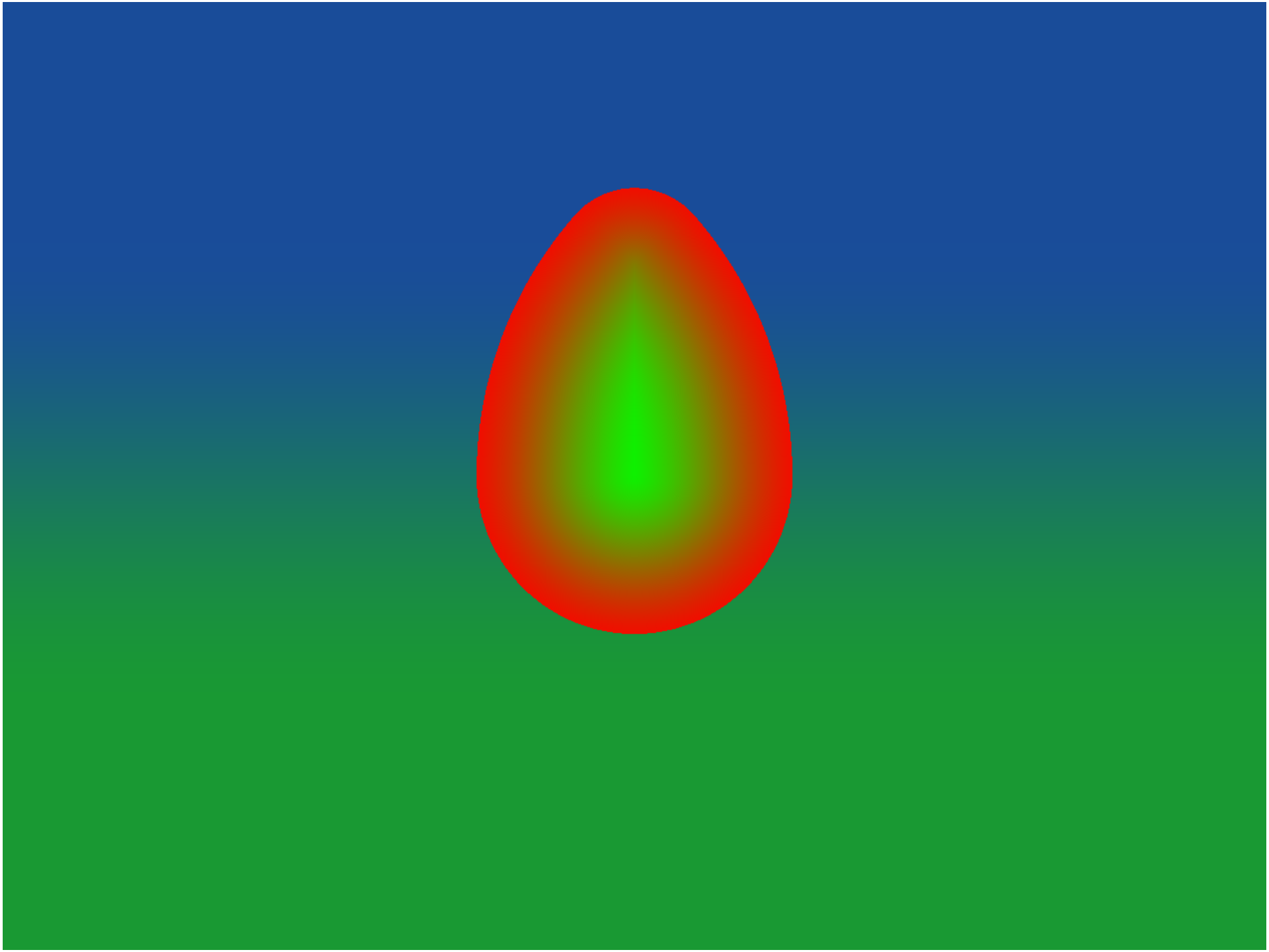
Indykator zbioru

$$\mathbb{I}_A(\mathbf{x}) = \frac{1 - \operatorname{sign}(\operatorname{sdf}_A(\mathbf{x}))}{2}$$



Maska

$$f(\mathbf{x}, \mathbf{c}) = \mathbb{I}_A(\mathbf{x}) \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix} + (1 - \mathbb{I}_A)(\mathbf{x}) \cdot \mathbf{c}$$



Funkcje okresowe

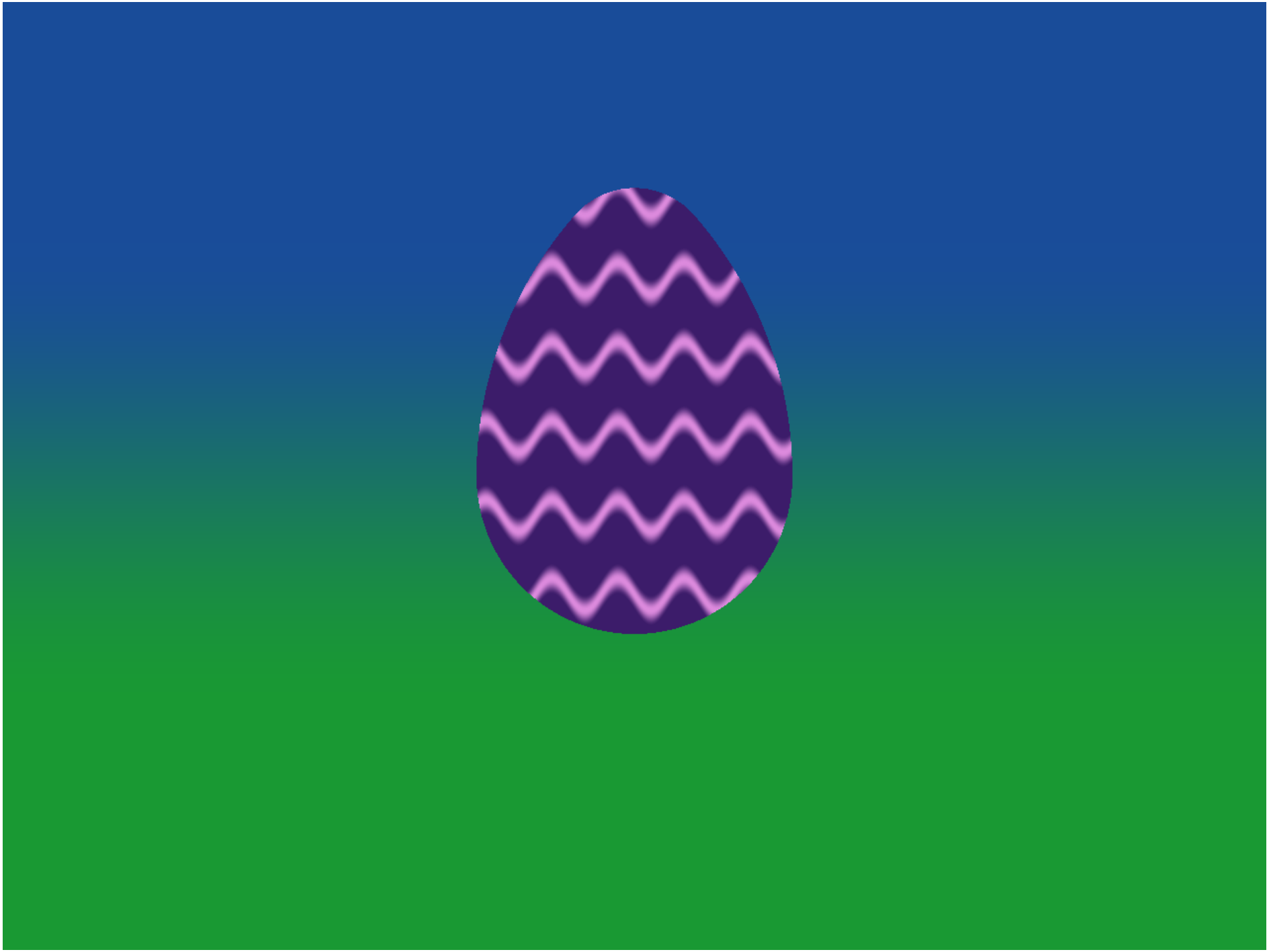
$$f(\mathrm{fract}(\mathbf{x}), \mathbf{x})$$





Zniekształcenia domeny

$$f(g_x(x), g_y(y), \mathbf{c})$$





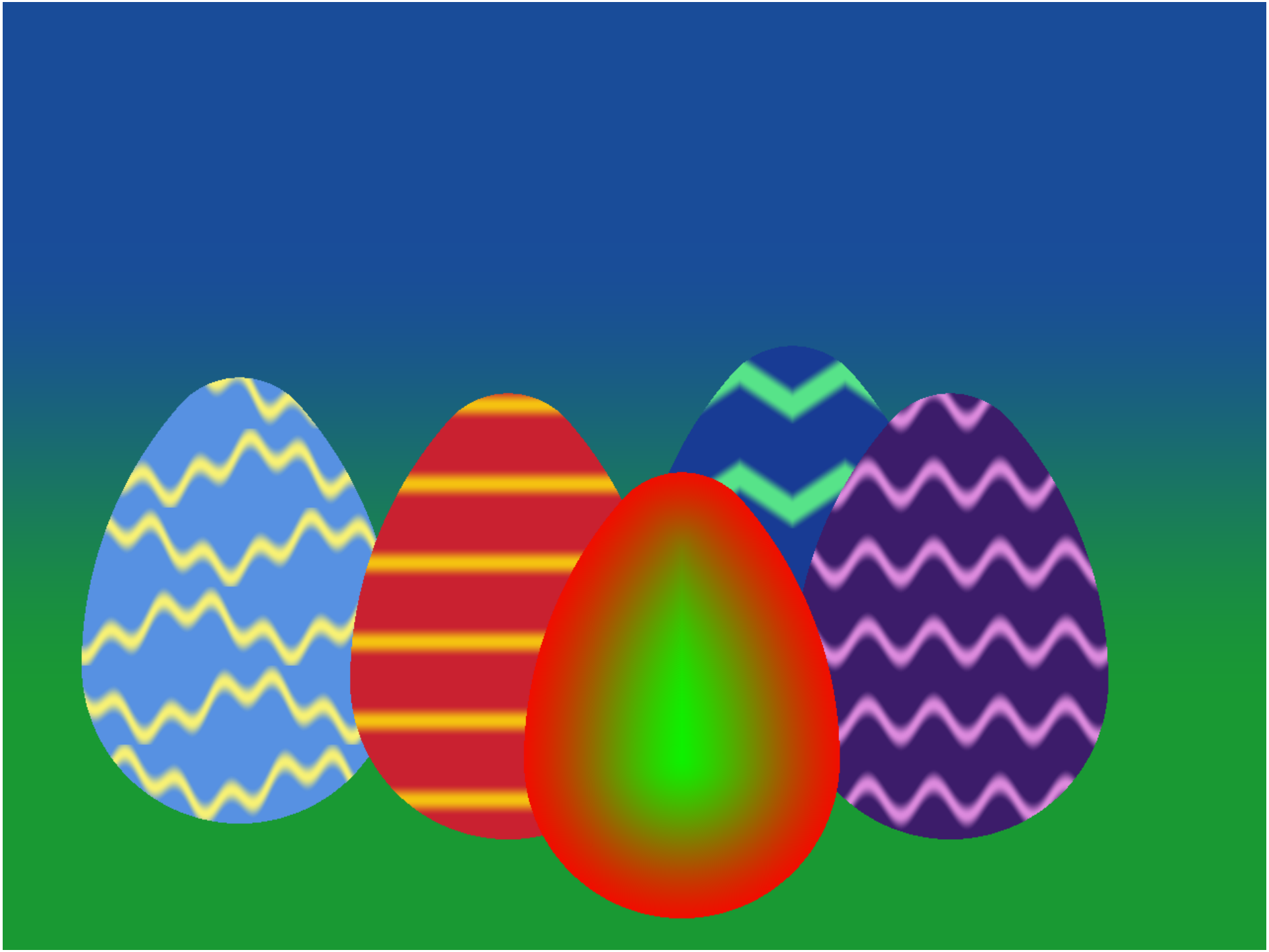
Transformacja

$$f(\text{transform}(\mathbf{x}, \mathbf{t}, \varphi, \mathbf{s}), \mathbf{c})$$

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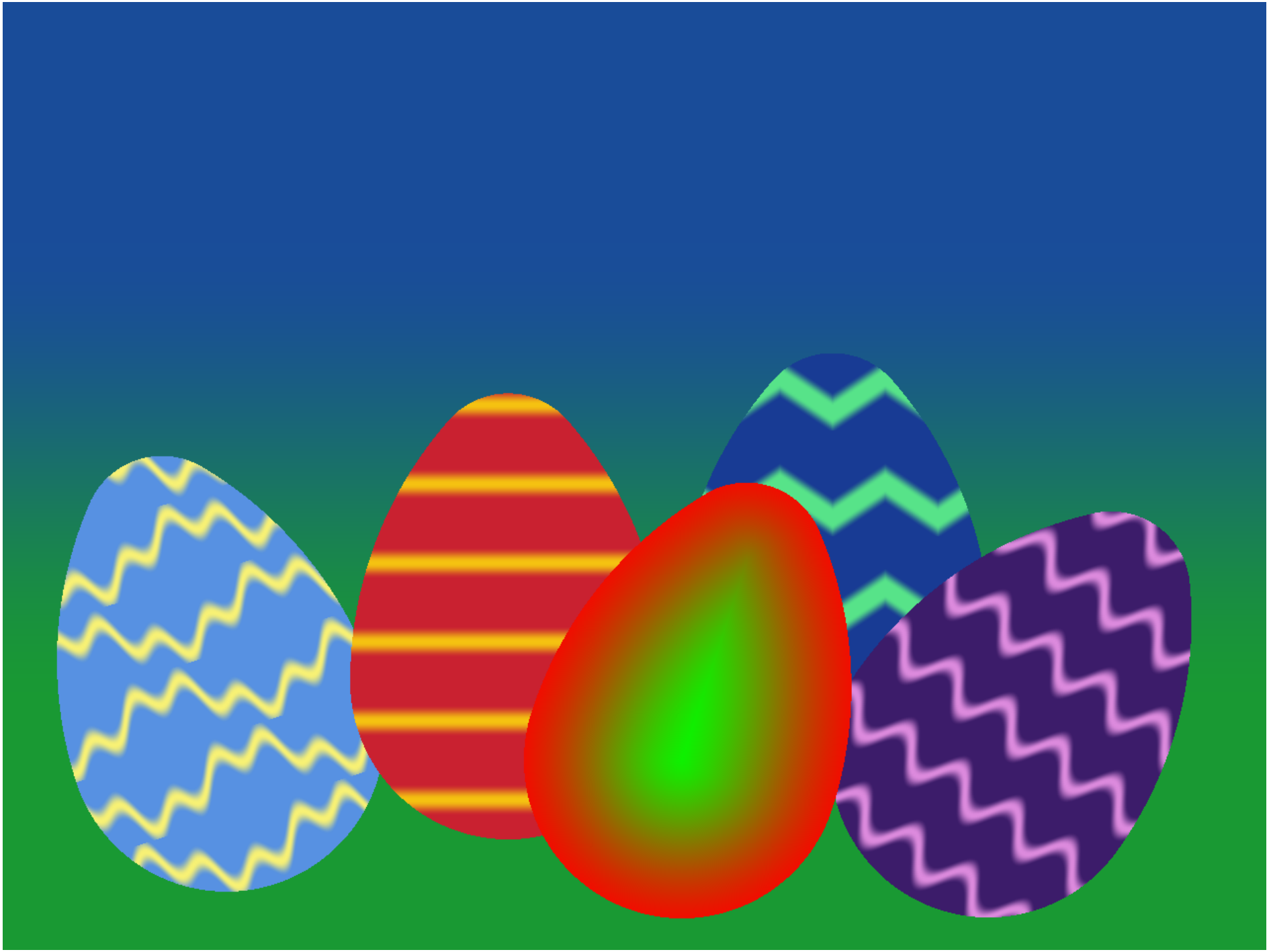
$$\begin{aligned} \text{transform}(\mathbf{x}, \mathbf{t}, \varphi, \mathbf{s}) &= \\ &= \qquad\qquad (\mathbf{x} - \mathbf{t}) \end{aligned}$$



Transformacja

$$f(\text{transform}(\mathbf{x}, \mathbf{t}, \varphi, \mathbf{s}), \mathbf{c})$$

$$\begin{aligned} \text{transform}(\mathbf{x}, \mathbf{t}, \varphi, \mathbf{s}) &= \\ &= \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \cdot (\mathbf{x} - \mathbf{t}) \end{aligned}$$



Transformacja

$$f(\text{transform}(\mathbf{x}, \mathbf{t}, \varphi, \mathbf{s}), \mathbf{c})$$

$$\begin{aligned}\text{transform}(\mathbf{x}, \mathbf{t}, \varphi, \mathbf{s}) &= \\ &= \begin{bmatrix} \frac{1}{\mathbf{s}_x} & 0 \\ 0 & \frac{1}{\mathbf{s}_y} \end{bmatrix} \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \cdot (\mathbf{x} - \mathbf{t})\end{aligned}$$




WESOŁEGO
JAJKA